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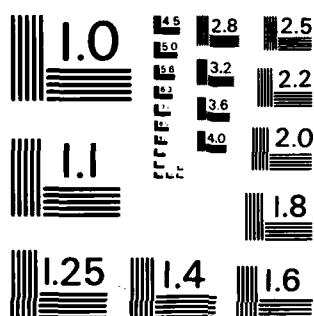
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A Procedure-Based Approach to Human Information Processing Models<sup>1</sup>

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## ABSTRACT

An approach is suggested for modeling human processing time in routine tasks. The existence of mental processing methods, or procedures, is presumed and the approach uses information theoretic concepts to develop a functional relationship between task variables and processing time for a given procedure. The resulting model contains parameters that must be estimated using processing time data. In addition to considering the single-procedure model, the modeling framework is extended to include situations where multiple procedures are used in an alternate fashion. The information theoretic framework provides a specific model form for the extra time required for switching to, or activating, a procedure.

The modeling approach is tested experimentally in two ways. First, a single procedure task is devised for which a model is developed. Second, a multiple procedure task is devised to test the model for switching. Experimental results in both cases give evidence in support of the approach as a method for describing task processing time in terms of task variables.

## I. INTRODUCTION

A major consideration within the approach to organization design in [1] is the development of an information processing model for each organization member. Basically, such a model is a description of human behavior at a particular information processing task, and has two components: an input/output map and an induced workload. This paper suggests a framework that might be appropriate for developing such a model, in the context of one particular class of information processing tasks. The class includes those tasks for which humans specific mental procedures, either naturally or due to prior training, and for which processing time is a meaningful measure of workload.

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Essentially, the approach uses concepts from information theory as an intermediate step in arriving at a model that associates task situation variables to processing time. The approach is related to previously developed information theoretic models of human behavior and represents a generalization of those models. In particular, the framework is designed to accommodate those situations where (a) one of several mental processing programs, or procedures, can be used to complete an information processing task, and (b) additional workload is incurred when the human switches among procedures.

Since the approach is essentially a descriptive one, its viability is closely related to whether it gives useful characterizations of human behavior. Several experimental tests have been conducted to investigate whether this is in fact the case. Because the tests have been conducted in a limited context, no general conclusions can be drawn. However, the results obtained do provide support for modeling approach.

It is important to emphasize that the primary intent of this paper is to document a particular approach to human modeling that is in an early stage of development. Thus, even though the model framework is described in general terms, its general applicability is largely untested. Further development of the approach beyond that which is reported in this paper is necessary before definite conclusions regarding its viability can be reached.

This paper is organized as follows. The next section describes the modeling approach. Fundamental premises are stated, and basic concepts from information theory that underly the approach are reviewed. Next, the synthesis of these concepts into a processing time model for a single procedure is made, followed by the extension to the more general case where multiple procedures are used. Section three documents experimental work that has been conducted to test the validity of certain features of the modeling approach. Finally, section four summarizes the paper.

## II. MODELING APPROACH

### 2.1 Underlying Concepts

Sanders [2] identifies a number of viewpoints that have been taken when modeling human behavior, including the view of humans as limited capacity processors. A fundamental premise in several approaches based on this view is that humans accomplish information processing tasks using "programmes". A program, or procedure, is a sequence of mental processing steps that are executed as a unit. Such a view is particularly appropriate in connection with a routine task at which a human has had much practice. Completing the task in this situation is then simply a matter of exercising the program that the human has developed for that task.

Another premise that is (according to Sanders) often associated with a limited capacity model is that mental processing resources are allocated in an all or nothing fashion to complete a task. As a consequence, processing load is directly related to observed processing time. Tasks that require more time to complete have required more processing resources, and hence have a higher workload.

These two premises are used as the basis for the modeling approach described in this paper. Procedures are taken as the basic unit and building block for describing human behavior at routine tasks. Procedures are modeled and distinguished by their input/output characteristics and by the time required for their execution. Moreover, the actual execution of a procedure in a given instance is often affected by current conditions that characterize the task, i.e. by the values of task variables. The focus of discussion in this paper is to develop an approach whereby procedure execution time can be expressed in terms of task variables. The development begins with consideration of information theoretic system models.

## 2.2 Information Theoretic Procedure Model

Analytic characterization of a general system of variables in information theoretic terms has been suggested by Conant [3]. The underlying principles used in developing such a characterization are discussed in the following paragraphs as they apply to the modeling of a human information processing procedure.

Consider the system shown in Figure 1. The system has an input  $x$  and

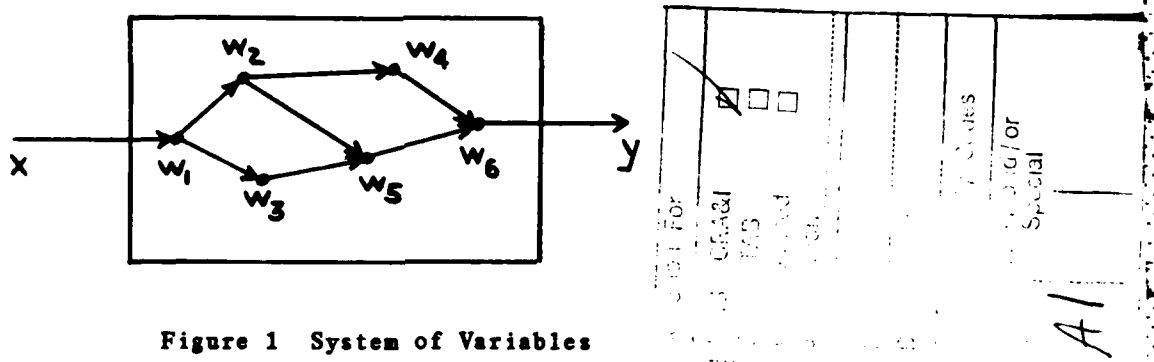


Figure 1 System of Variables

an output  $y$ . In addition, there are six variables  $w_i$  that are internal to the system. These represent intermediate steps in the processing of input into output, and the interconnecting arrows indicate the immediate dependencies of one internal variable on the others in the system. For the system shown, the input  $x$  is first transformed, or processed, into the variable  $w_1$ . The latter is in turn transformed, in independent operations, into variables  $w_2$  and  $w_3$ . The variable  $w_4$  depends on the values of  $w_2$  and  $w_3$ , and so on until the value of the output is determined. The specific input  $x$  that is received is determined according to the distribution on  $x$  values,  $p(x)$ .

Conant defines the total processing activity, denoted by  $g$ , in such a system to be the sum of the entropies of each internal variable. For the system in Figure 1

$$g = \sum_{i=1}^6 H(w_i) \quad (1)$$

where  $H(w_i)$  denotes the entropy of internal variable  $w_i$ .  $H(w_i)$  is typically expressed in bits and is evaluated from the probability distribution  $p(w_i)$  [4]:

$$H(w_i) = - \sum_{w_i} p(w_i) \log_2 p(w_i) \quad (2)$$

The usual information theoretic interpretation of the quantity  $H(\cdot)$  is that of uncertainty. A variable  $w_i$  that takes only one value has zero uncertainty as measured by  $H(\cdot)$ . As the probability distribution on values of  $w_i$  becomes more equalized, however, the uncertainty in the value it will take on a particular outcome increases.

In a system of variables, the processing of input into output requires the determination of value for internal variables in step by step fashion, beginning at the input and ending with the output of the system. If processing activity is taken as the resource used to resolve the uncertainty in each internal variable's value, then the amount of this resource required for the system is directly related to the quantities  $H(w_i)$ . This is the basis for defining the total activity of a system as in eq.(1).

A particular system that will be of interest in the sequel is that of a channel, which is a system with a single internal variable as shown in Figure 2. The basic operation of the system is to receive inputs  $x$  and to pass them through as outputs  $y$ . Total activity for this system is

$$g = H(w_1) \quad (3)$$

The usual information theoretic characterization of a channel, however, is that of the relatedness of input to output, or mutual information  $T(x:y)$  [4]:

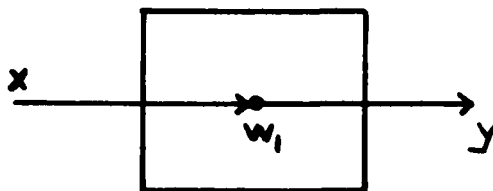


Figure 2 Single Variable System - A Channel

$$T(x:y) = H(y) - H_x(y) = H(x) - H_y(x) \quad (4)$$

The quantity

$$H_x(y) = - \sum_x p(x) \sum_y p(y|x) \log_2 p(y|x) \quad (5)$$

is the conditional uncertainty in  $y$ , given the value of  $x$ . Eq.(4) measures the degree to which channel output is related to channel input. If  $x$  determines  $y$  uniquely and  $x$  can be uniquely inferred from  $y$ , then the channel is error-free and  $T(x:y)$  is at a maximum:

$$T(x:y) = H(y) = H(x) \quad (6)$$

Moreover, the internal system variable must be an identity operator in the case of an error-free channel, which means that

$$g = H(w_1) = H(x) = T(x:y) \quad (7)$$

In other words, total activity for an error-free channel is identically equal to the channel throughput, which means that system processing activity is entirely devoted to faithfully reproducing the input  $x$  as the output  $y$ .

In a channel that is subject to error, however, a different relationship between total activity and mutual information occurs. In this case the mapping between inputs and outputs is no longer one-to-one.



Two other possibilities exist for the processing that occurs within the system (at the internal variable): either a many-to-one mapping is realized at  $w_1$  (loss of information about  $x$ ) or a one-to-many (probabilistic) mapping is made at  $w_1$  (spurious information, i.e. noise, in  $y$ ). In either case, there is additional processing activity within the system that is not reflected in system throughput. Thus for the situation of an imperfect channel it is true that

$$g = H(w_1) \geq T(x:y) \quad (8)$$

Conant's characterization of system activity carefully categorizes types of processing activity and situations that lead to relationships such as eq.(8). For present purposes, it is sufficient to note that (a) an error-free channel is equivalent to a single variable system in the sense that channel throughput and system total activity are identical, and (b) that such systems are a subset of those that can be characterized using internal variables and total activity.

The characterization of a system in terms of its total activity using internal variables has several features of interest in the present context. First, since internal variables correspond to intermediate steps in processing, total processing activity will tend to increase with the number of processing steps. Secondly, since values of internal variables generally depend on input values, the distributions  $p(w_1)$  will vary as the characteristics of the input vary. In other words, total activity is a function of  $p(x)$ .

The two properties cited above suggest that total activity might be adapted as a measure of procedure workload. The correspondence is as follows. A system of variables is a processor of inputs into outputs. To the extent that this processing is well-defined and recognizable as a unit, it can be associated with the information processing that a human accomplishes when executing a procedure. To each procedure there can be assumed to correspond a system of variables. By associating system processing activity with human processing load, the quantity  $g$  can lead to

a characterization of task workload. This correspondence is pursued and further developed in the next section.

### 2.3 Processing Time and Total Activity

The discussion in the previous section has suggested total activity as a possible workload measure. In the present context, however, there are difficulties associated with defining internal variables. This limits the usefulness of total activity as a direct measure of workload. Despite the limitation, characterization of a procedure's total activity can be used as an intermediate step toward characterizing procedure workload in terms of processing time. The following paragraphs describe how this might be accomplished. The discussion proceeds developmentally by first attempting to use total activity directly. At the point where the limitations in doing so become apparent, a modification is made that eventually leads the discussion to the modeling approach of interest.

#### Synthesis of Concepts

Information theoretic models of human information processing are not new, however. Considerable research activity was once focused on investigating the validity of modeling the human as a limited capacity channel (for a review see [5]). Some degree of success was realized and a basic relationship between mean response time ( $\bar{t}_p$ ) and the mutual information between inputs and outputs  $T(x:y)$  emerged as valid for a variety of situations:

$$\bar{t}_p = a + b \cdot T(x:y) \quad (9)$$

In words, eq.(9) characterizes the human, in an information processing situation, as a fixed delay in series with an information transmission channel, where the channel is assumed to be of limited capacity. A key consideration in previous work was the characterization of human capacity in information theoretic terms. There was much less success at doing

this, however, perhaps because the full amount of human processing activity was not reflected in measured channel throughput.

The relationship of eq.(9) can be extended in a natural way to include total activity. In previous investigations the usual situation in which eq.(9) was shown to be valid was one that involved a pure transmission task (e.g. see [6] and [7]). That is, for each possible stimulus there was a particular response that the subject was supposed to make. In information theoretic terms, the task was to transmit each input, without error, through a channel, so that it would appear as the desired distinct output, though perhaps in a different form. As discussed earlier, however, error-free channels are a specialized system of variables, in which information transmission  $T(x:y)$  also happens to be the total activity  $g$ . Using this point of correspondence, one can re-interpret the characterization of eq.(9) as a relationship between average processing time and total activity, which just happens to have been tested only for pure transmission systems. That is, a more general form for the relationship expressed in eq.(9) might be

$$\bar{t}_p = a + b \cdot g \quad (10)$$

Eq.(10) models the human, in an information processing task, as a fixed delay in series with an information processing system. More precisely, the characterization in eq.(10) is for an information processing system that corresponds to a single procedure. It will be generalized in the sequel to include the case of multiple procedures. Before doing so, however, consideration will be given to further developing the model for a single procedure.

#### Construction of Procedure Model Based on Total Activity

Consider now the construction of a processing time model based on total activity. Suppose that a given task is known to be accomplished by the execution of a single procedure and that this procedure can be readily activated and observed. Conceptually, the modeling might be done by

identifying the internal variables of the procedure and their relationship to each other. From this structure, the probability distributions on each internal variable could be determined, assuming that the distribution on inputs  $p(x)$  is known. Then the sum in eq.(1) could be calculated to obtain  $g$ . This knowledge, together with the observed average processing times, could then be used to estimate  $a$  and  $b$  in eq.(10), assuming that data corresponding to more than one distribution  $p(x)$  were available.

The approach described above is not feasible, however. The basic difficulty is with the identification and definition of internal variables. In the development of [3], it is assumed that one can define in an absolute sense, or at the very least in a self-consistent sense, the internal variables of a system. This is necessary not only for calculation of total activity within a system, but also for purposes of meaningful comparison of different systems. That is, total activity of one system measured in bits should be comparable to the total activity of another system when measured on the same scale. Using total activity to describe a human information processing procedure requires the identification of variables that are internal to the human. Even if such variables exist, they are not observable, and there is consequently little basis for their definition in a consistent manner. A modification in the approach is therefore necessary in order to use total activity to develop a processing time description.

While the set of internal variables unique to a particular procedure may be impossible to determine, it is still plausible to presume the existence of some intermediate steps in processing within the execution of a procedure. For example, such steps could be postulated in association with comparison tests made in processing or with an obvious data aggregation operation that is required to complete a task. A system of internal variables could then be established based on the delineation of these processing steps. However, the variables would only be representative of those that might exist, and the question arises as to what extent the value obtained from the summation of eq.(1) would reflect actual total activity. In other words, by discarding the notion that

internal variables can be defined in an absolute way, one also eliminates a consistent and uniform scale. It is not apparent how much weight should be attached to the uncertainty of each "representative internal variable" relative to the others defined.

A uniform scale is available, however, in the form of processing time. Using the representative internal variables as a set of a basis variables, the processing time associated with procedure execution can be projected onto this basis, which thereby establishes relative weightings on the variables. That is, the foregoing discussion has suggested that the total activity characterizing a given procedure can be represented as

$$g = \sum_{i=1}^N \bar{c}_i \cdot H(w_i) \quad (11)$$

where  $w_i$  is the  $i^{\text{th}}$  representative internal variable and  $\bar{c}_i$  is the relative weighting (as yet unspecified) to be attached to variable  $w_i$ . From eq.(3), the average processing time then becomes

$$\bar{t}_p = a + b \sum_{i=1}^N \bar{c}_i \cdot H(w_i) \quad (12)$$

Note that  $\bar{t}_p$  still depends on input characteristics because variation in  $p(x)$  will in general change the values of  $H(w_i)$ .

In eq.(12) the parameters  $a$ ,  $b$ , and  $\{\bar{c}_i\}$  are unspecified and must be estimated using the processing time observed when the task that exercises the procedure is executed. Given this approach, it is not possible to uniquely determine  $b$  and  $\{\bar{c}_i\}$ , however. Therefore let  $c_i = b\bar{c}_i$ , ( $i = 1, \dots, N$ ) and  $a = c_0$ . Eq.(12) then becomes

$$\bar{t}_p = c_0 + \sum_{i=1}^N c_i \cdot H(w_i) \quad (13)$$

Estimating values for the parameters  $c_i$  requires that  $p(x)$  be varied over its possible range. Values of  $H(w_i)$  can then be computed, and values of  $\bar{t}_p$  observed. From this data parameter estimates can be made. The result

is a model that relates the input probability distribution to the average time required to process an input.

### Example

As an example of how the modeling approach might be carried out, consider the task situation shown in Figure 3. The task requires the

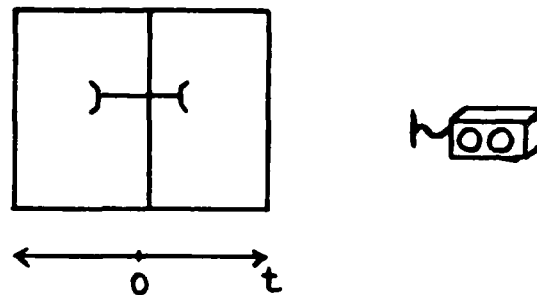


Figure 3 Task Situation

observation of the horizontal location of the pattern's midpoint ( $x$ ) and then a decision whether or not it exceeds the threshold  $t$ . Two internal variables are defined, one corresponding to the pattern observation and the other corresponding to the comparison test:

$$\begin{aligned} w_1 &= x \\ w_2 &= x > t \end{aligned} \quad (14)$$

Substituting (14) into eq.(13) yields

$$\bar{t}_p = a + b \cdot g = c_0 + c_1 \cdot H(w_1) + c_2 \cdot H(w_2) \quad (15)$$

Suppose  $x$  is normally distributed with mean  $m$  and variance  $\sigma^2$ . Then [4]

$$\begin{aligned} H(w_1) &= \int_{-\infty}^{\infty} p(w_1) \log_2 p(w_1) dw_1 \\ &= \log_2(2\pi\sigma) \end{aligned} \quad (16)$$

and

$$\begin{aligned}
 H(w_2) &= - \sum_{w_2} p(w_2) \log_2 p(w_2) \\
 &= H(\delta)
 \end{aligned}
 \tag{17}$$

where

$$\delta = \delta(t, m, \sigma) = p(x < t) = \int_{-\infty}^t p(w_1) dw_1
 \tag{18}$$

Note that for fixed  $t$ , the value of  $\delta$  depends on the values of  $m$  and  $\sigma$ , which are characteristics of  $p(x)$ .  $H(\delta)$  is the uncertainty of a variable which takes two possible values, one with probability  $\delta$  and the other with probability  $1-\delta$ , as shown in Figure 4. By appropriate substitution,

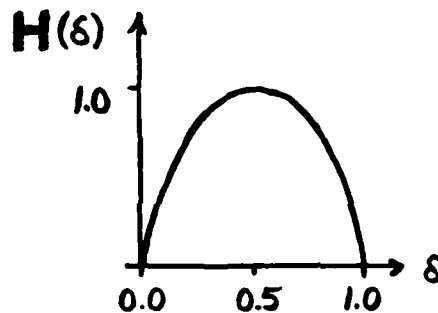


Figure 4 Uncertainty in Binary Random Variable

eq.(15) becomes

$$\bar{t}_p = c_0 + c_1 \cdot \log_2(2\pi\sigma) + c_2 \cdot H(\delta)
 \tag{19}$$

The right-hand expression in eq.(19) has three unknown parameters ( $c_i$ ). It depends also on the known parameters  $m$  and  $\sigma$ . Values of  $c_i$  can be estimated from processing time data collected for various values of  $m$  and  $\sigma$ .

The modeling approach outlined above is one of postulating a set of internal variables for a given procedure and then projecting the total processing time of the procedure onto the respective uncertainties of those variables. Careful selection of a proper set of "basis" variables

is therefore necessary in order to adequately account for the processing time associated with a procedure. As a guideline for choosing variables, consider that a procedure can be regarded as something that modifies a distribution on inputs into a distribution on outputs. This modification is done through intermediate steps, to which are also associated distributions. Major changes in the probability distribution as it moves from input to output are therefore good candidates for internal variables, since a change in the distribution on a variable's possible values will generally imply a change in the uncertainty associated with it. Typical steps in processing that might be identified and used to generate internal variables include observation of external stimuli, fusion of information, elimination of irrelevant information, and comparison of one value with another.

The foregoing exposition of the modeling approach has treated internal variables in a general way. Note, however, that the example given has used only variables that are also observable externally. Subsequent discussion and experimental testing in this paper will also involve only observable internal variables, i.e. ones that are also input or output variables. The issue of whether variables that are truly internal can be defined and used in the context of this modeling approach will not be addressed in this paper. This issue is of importance, however, and must be investigated in order to assess the general applicability of the approach.

#### 2.4 Multiple Procedure Models with Switching

Suppose now that an information processing task is such that more than one procedure is used, and that the human is required to switch from one procedure to another frequently. This might occur in a situation where an overall task is really two separate, interspersed tasks that require their own respective type of processing. It might also occur in the situation where more than one way has been provided to accomplish a given task. In general, switching from one processing method to another



will require an amount of re-orientation, which itself uses processing resources. Given the present assumptions about processing resources and processing time, switching procedures frequently will require additional processing time. Switching overhead is an intuitive concept, and has been observed and noted previously [8].

The following paragraphs describe the extension of the single procedure model to situations where multiple procedures are used in an alternate fashion. Briefly, consideration of the total activity of a system with switching leads to a specific model form for the processing activity required for switching. This model is then used to generalize the single procedure model structure introduced in the previous section.

#### Processing System with Switching

Consider the system of variables shown in Figure 5. It consists of

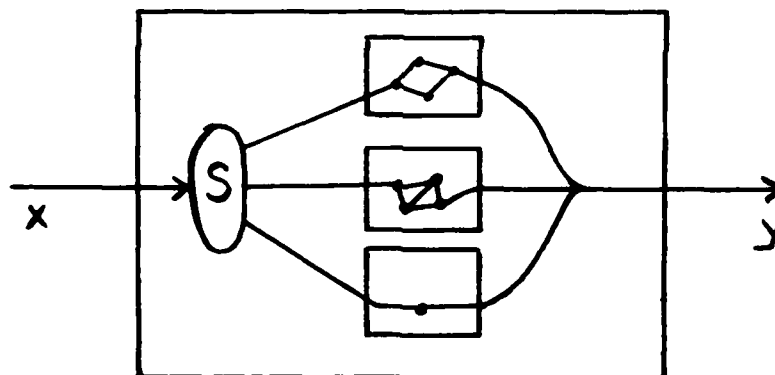


Figure 5 System with Switching

three subsystems that are connected by a switch (S). Inputs  $x$  arrive and, depending on the switch, are routed to one of the three subsystems. The input is then processed by the chosen subsystem and an output is produced, which is also the output  $y$  of the overall system. System operation is such that only one input is processed at a time, i.e. there is no parallel processing possible.

The total activity of the system shown in Figure 5 is, by definition,

the sum of the uncertainties  $H(\cdot)$  of each internal variable. As a matter of definition, there are no internal variables associated with the switch.<sup>2</sup> Thus the overall total activity can be written as a double sum over the internal variables  $w_{ij}$ , where  $j$  indexes the internal variables in subsystem  $i$ :

$$S = \sum_{i=1}^3 \sum_j H(w_{ij}) \quad (20)$$

If one regards each subsystem as a system of variables unto itself, then each inner summation (one for every  $i$ ) in eq.(20) represents the total activity of that subsystem. However, because of the operation of the overall system, it is not true that the value obtained for

$$\sum_j H(w_{ij}) \quad (21)$$

for a particular  $i$  is the same that would be obtained if subsystem  $i$  were the entire system. Due to the switching present, subsystem  $i$  is not necessarily used to process each input; in fact it may not be used at all. Thus its contribution to the total processing activity of the overall system must be related in some way to its relative frequency of use and also to the characteristics of those inputs that it does process. The particular expression for this contribution is in fact given by [9]

$$\sum_j H(w_{ij}) = p(i) \cdot g_i(p(x|i)) + \bar{u}_i \cdot H(p(i)) \quad (22)$$

In eq.(22),  $g_i$  is the total activity of subsystem  $i$ , which has been written as a function of the particular distribution on  $x$  values that it actually processes. Note that this may be different than  $p(x)$  due to the action of the switch. The quantity  $p(i)$  is the fraction of system  $i$ 's use. In the second term on the right-hand side,  $\bar{u}_i$  is the number of

<sup>2</sup>In general, this need not be the case when building an information theoretic model using the present framework. However, for purposes of adaptation to a processing time model, the assumption made here is one of convenience.

internal variables in subsystem  $i$  and  $H(\cdot)$  is the uncertainty in a binary random variable (see Figure 4).

The latter term in eq.(22) represents processing activity that is additional to the activity actually required to process an input into an output. It is interpreted as the processing activity due to switching or, in other words, the activity required to activate the subsystem. In general, the form of  $H(\cdot)$  is such that an often-used subsystem can have the same switching overhead as a seldom-used one. In the latter case, seldom usage means seldom activation; in the former case, frequent usage means that the subsystem may be used to process successive inputs and thus would not need to be re-activated. In particular, if the subsystem is either used exclusively or not at all the overhead for switching is zero.

Substitution of eq.(22) into eq.(20) gives an expression for the overall system activity;

$$g = \sum_{i=1}^3 \left[ p(i) \cdot g_i(p(x|i)) + \bar{a}_i \cdot H(p(i)) \right] \quad (23)$$

Eq.(23) generalizes in an obvious way to a situation where  $N$  subsystems are present.

There are two essential features of eq.(23) for present purposes. The first is that the overall system processing activity is obtained as a weighted sum of individual subsystem total activities, where total activity is computed given that subsystems are active. A second feature is the particular model derived for the processing activity due to switching among subsystems. These two features will be used to adapt eq.(23) to obtain a processing time model for human information processing in a situation where switching among procedures is evident.

#### Processing Time and Procedure Switching

Having analyzed and developed a model, in information theoretic terms, for a system with switching, it is possible now to consider how

that model can be adapted to a description of human information processing in situations where multiple procedures exist. Figure 6 shows an

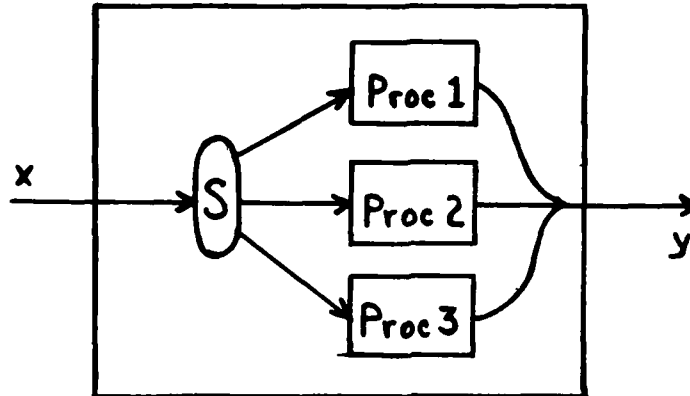


Figure 6 Multiple Procedure Information Processing Model

information processing model that consists of three procedures, which is complementary to the system of Figure 5. For each input  $x$ , a procedure must be selected for processing  $x$  into  $y$ . This selection may be made at the discretion of the human or it may be dictated by the particular  $x$  received. In either case, the task is such that each procedure will in general be used on only a fraction of the inputs. It is desired to adapt the model of eq.(22) to the situation shown in Figure 6.

To begin, recall the processing time model for a procedure derived earlier. Adding a subscript  $i$  to designate the particular procedure under consideration, eq.(10) becomes

$$\bar{t}_{pi} = a_i + b_i \cdot g_i \quad (24)$$

In the present context, eq.(24) characterizes the processing time of procedure  $i$ , given that it is used. Neglecting the effect of switching and assuming that procedures retain their characteristics as a unit when they are part of a multi-procedure situation, a model for the overall processing time for the structure in Figure 6 might be

$$T_p = \sum_{i=1}^3 p(i) [a_i + b_i \cdot g_i] \quad (25)$$

Now consider the question of how to adapt the expression for processing activity due to switching to a processing time model. From the basic assumption that use of processing resources corresponds directly to use of processing time, it appears that the average additional processing time due to switching, denoted  $\bar{t}_s$ , is of the form

$$\bar{t}_s = d \sum_i \bar{a}_i \cdot H(p(i)) \quad (26)$$

or possibly of the form

$$\bar{t}_s = \sum_i d_i \cdot \bar{a}_i \cdot H(p(i)) \quad (27)$$

where  $d$  and  $\{d_i\}$  are unspecified constants. Recall that  $\bar{a}_i$  designates the number of internal variables in subsystem  $i$ . As discussed earlier, it is not possible to absolutely define the internal variables of a procedure; therefore  $\{\bar{a}_i\}$  are not known. Operationally, however, they represent a scale factor on the function  $H(\cdot)$  that measures the contribution to switching activity of subsystem  $i$ . This characteristic can be retained in the processing time model, although it is not possible or necessary to identify  $\{\bar{a}_i\}$  and  $\{d_i\}$  (or  $d$ ) individually. Therefore, let

$$\bar{t}_s = \sum_i \alpha_i \cdot H(p(i)) \quad (28)$$

where  $\{\alpha_i\}$  are parameters with units of time/bits. Adding eq.(28) to eq.(25) gives the full model for a multi-procedure situation.

$$T_p = \sum_{i=1}^3 \left[ p(i) \cdot \bar{t}_{pi}(p(x|i)) + \alpha_i \cdot H(p(i)) \right] \quad (29)$$

In eq.(29),  $\bar{t}_{pi}$  designates the model for procedure  $i$  derived earlier using the information theoretic approach and is evaluated using the relevant distribution on inputs  $x$ .

## 2.5 Summary

The preceding discussion has outlined an approach to modeling human information processing in terms of processing time. The approach is derived from the information theoretic characterization of systems in terms of their total processing activity. By associating system processing activity with human information processing resources, an expression was derived for the time required by a human to execute a procedure to accomplish a given information processing task. The expression given in eq.(13) represents a generalization of the classical information theoretic model of human behavior. In particular, the existence is postulated of internal variables that represent intermediate steps in the processing of inputs to outputs. Since these variables are themselves unobservable, the procedure model is derived by projecting processing time onto a basis set of assumed internal variables.

The information theoretic view of a system has also suggested a model for the situation where multiple procedures are used. Eq.(29) summarizes the multi-procedure processing time model. It includes special terms that represent additional processing time required for switching to, or activating, procedures.

The models given in eq.(13) and eq.(29) have been formulated with specific regard to their eventual use as descriptions of human behavior. The next section tests the modeling approach in this regard by applying it to specific tasks.

## III. EXPERIMENTAL EVIDENCE

To test the modeling approach described in the previous section, two sets of experiments have been devised. This section presents and

discusses results from these experiments. The first set is designed to examine the validity of the modeling approach for a single procedure and the second set is designed to test the form of the switching model as it is given by the expression in eq.(28). The discussion that follows is organized into two parts, one for each set of experiments.

### 3.1 Test of Procedure Model

#### 3.1.1 Information Processing Task I

##### Description

As a test of the approach to representing procedure processing time, consider again the task situation used as an example in section 2.3, which is shown here as Figure 7. On each trial, a value of  $x$  is generated

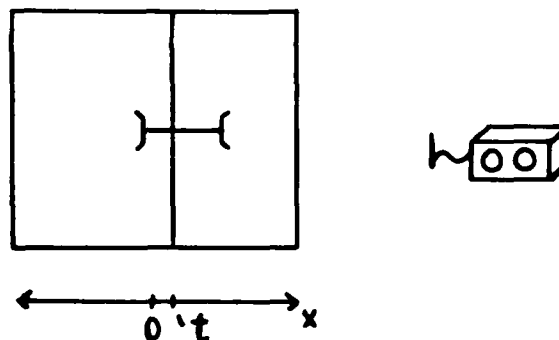


Figure 7 Single Procedure Task Situation

randomly according to a normal distribution:

$$p(x) \sim N(m, \sigma^2) \quad (30)$$

and displayed as the horizontal displacement of the midpoint of the "pattern" that is illustrated in the figure. The subject must judge whether the observation is left or right of the vertically displayed threshold  $t$ . He registers this judgement by depressing one of two horizontally-arranged buttons.

### Procedure Model

It is desired to describe the processing time required to do the above task for a range of threshold positions and for a range of input characteristics. Assuming that a human, after sufficient practice, will develop a mental processing method for doing the task, the situation is one where the procedure modeling approach can be tested.

To begin, define two representative internal variables (which also happen to be observable) for the pattern processing procedure, one corresponding to the pattern observation and the other corresponding to the threshold comparison test. These are given by  $w_1$  and  $w_2$ :

$$\begin{aligned} w_1 &= x \\ w_2 &= x \begin{matrix} > \\ < \end{matrix} t \end{aligned} \quad (31)$$

As discussed in 2.3, the form of the processing time model derived via information theoretic considerations is given as

$$\bar{t}_p = c_0 + c_1 \cdot H(w_1) + c_2 \cdot H(w_2) \quad (32)$$

In this particular situation, eq.(32) specializes to (see the development preceding eq.(19))

$$\bar{t}_p = c_0 + c_1 \cdot \log_2(2\pi\sigma) + c_2 \cdot H(\delta) \quad (33)$$

where

$$\delta = \Phi \left( \frac{t - m}{\sigma} \right) \quad (34)$$

and  $\Phi(\cdot)$  is the cumulative distribution function for a  $N(0,1)$  random variable. Eq.(33) contains three parameters that must be identified using experimental data.



### 3.1.1 Test of Model

#### Setup and General Procedure

To test the model of eq.(33), experimental runs were carried out using the following setup. A subject was placed before a CRT that continuously displayed the square border shown in Figure 7. The border was dimensioned to be 10 units on a side (approximately 6 inches), with the upper right and lower left corners at the coordinates (5,5) and (-5,-5), respectively. The mechanical response buttons were mounted in a hand-held panel.

On each trial in an experimental run, a psuedo-random value of  $x$  was generated according to the distribution established for that run. These values became the horizontal position of the pattern midpoint. In order to make the task slightly more challenging, another pseudo-random number was generated from a zero mean distribution to obtain a vertical displacement of the pattern on each trial. Once pattern position was determined, the pattern and threshold were displayed simultaneously. The subject was instructed to respond as quickly as possible, but with certainty, as to the pattern's position relative to the threshold. After the response was made, the pattern disappeared and an 800 ms blanking period intervened before the next trial. To ensure that no pattern was impossibly hard because of the nearness of its midpoint to the threshold, a dead zone was established. Any value of  $x$  that fell within  $\pm \Delta x$  of  $t$  was adjusted so that it was exactly  $\Delta x$  units away. This occurred relatively infrequently, however, and less than 5% of the  $x$  values were so adjusted.

#### Experimental Tests and Results

For the set of experimental conditions used to test the procedure model, the value of  $m$  was set at zero. Furthermore, the vertical position of each pattern was established randomly according to a number drawn from

a  $N(0,1)$  distribution. The experimental conditions used were distinguished according to the values of  $t$  and  $\sigma$  assigned. Table 1 lists the

Table 1 Single Procedure Experimental Conditions

Condition #	Parameter Estimation					Validation		
	1	2	3	4	5	6	7	8
$\sigma$	0.5	2.0	1.0	1.5	2.0	2.0	0.5	1.0
$t$	0.0	0.0	1.56	1.75	1.5	0.5	0.45	1.28

$(t, \sigma)$  pairs that have been used and labels them with a condition number. After much practice in previous sessions, all eight conditions were tested within a single session, but in random order. The test at each condition consisted of a 50 trial run followed by a second run of 150 trials. During the first run, the subject was given the opportunity to adjust his mental method for the conditions. Presumably the second run would then represent a succession of identical executions of the same procedure.

Results for two sessions using the same subject<sup>3</sup> are shown in Table 2. Columns labeled  $\bar{t}_p$  contain observed mean response time data and columns labeled  $p_e$  list the observed error rate for the run. To test the model, the first five conditions were used to estimate values for the parameters  $c_i$  using a least squares procedure. Table 3 lists these estimates along with the bounds that represent a simultaneous 95% confidence interval on all three parameters.

Using the estimates for  $c_i$ , values of mean response time were computed for each condition and are listed in the column of Table 2 labeled  $\hat{t}_p$ . Since conditions 6-8 were not used to calculate  $c_i$ , the value

<sup>3</sup>The subject is the author. Due to the circumstances, it was not possible to train other subjects and collect data from them. However, even though the general procedure model form was known to the subject a priori, the specific relationship sought between test conditions and response time data is not readily apparent. Thus the extent to which data can be manufactured is not believed to be a major issue.

Table 2 Single Procedure Processing Time Results

Condition	Session 1				Session 2			
	$p_e$	$\bar{t}_p$	$\hat{t}_p$	95% Conf.	$p_e$	$\bar{t}_p$	$\hat{t}_p$	95% Conf.
1	0.047	282	285	-	0.027	285	286	-
2	0.060	247	249	-	0.020	234	239	-
3	0.047	239	239	-	0.033	238	243	-
4	0.060	229	237	-	0.013	234	235	-
5	0.073	238	239	-	0.033	232	233	-
6	0.067	233	247	$\pm 6.1$	0.027	241	238	$\pm 6.2$
7	0.080	264	272	$\pm 7.4$	0.040	278	277	$\pm 7.5$
8	0.040	255	245	$\pm 5.4$	0.040	248	247	$\pm 5.5$

Table 3 Procedure Model Parameter Estimates

Session	$\hat{c}_0$		$\hat{c}_1$		$\hat{c}_2$	
		95%		95%		95%
1	273	$\pm 18.4$	-18.0	$\pm 4.8$	41.4	$\pm 13.7$
2	295	$\pm 18.8$	-23.4	$\pm 4.9$	29.4	$\pm 14.0$

of  $\hat{t}_p$  for these conditions represents a prediction by the model that can be compared with observed data. For these conditions, a 95% confidence interval has been calculated as well and is given in Table 2.

### Discussion

For both sessions, the estimated parameter values provide a good fit to observed results for conditions 1-5. There is some degree of consistency evident across sessions, but the estimation process was observed to be sensitive to small changes in  $\bar{t}_p$  values. Thus while the observed  $\bar{t}_p$  values do not differ greatly for Sessions 1 and 2 in conditions 1-5, there is a somewhat greater discrepancy in the values of  $c_1$  across sessions.

The characteristics of conditions 1-5 were chosen so that the confidence intervals calculated for other conditions (6-8) would be small enough to give meaningful predictions. This has indeed been realized, as is apparent from comparison of the magnitudes of the 95% confidence intervals in Tables 2 and 3. For the second session, the predicted values given by  $\hat{t}_p$  in conditions 6-8 match quite well the observed values of  $\bar{t}_p$ . Session 1 results do not exhibit such agreement, however. Indeed, if the 95% confidence interval were used as the criterion for testing the model, one would readily accept the procedure model based on Session 2 results, but would reject it based on Session 1 results.

An explanation for this inconsistency is not readily apparent. One possibility is to question the reliability of the data from the first session based on the error rates observed. The values of  $p_e$  for session 1 are higher for each condition than those of session 2 by a few percent. Furthermore, the error rates for the second session are more typical of those observed for the subject in previous sessions. It may be concluded, therefore, that other processing mechanisms are present in Session 1 and that Session 2 is in some sense a more reliable test of the model. From the evidence recorded in Tables 2 and 3, however, a definite conclusion cannot be made. It is fair to conclude, however, that there is evidence in the experimental results that supports the procedure model.

### 3.2 Test of Switching Model

#### 3.2.1 Information Processing Task II

##### Description

To test the switching model, the information processing task shown in Figure 8 was devised. One of three possible "thresholds" is presented to the subject on each trial: a vertical line (a), a horizontal line (b), or a circle (c). In each situation, a dot is also displayed and the subject is to select one of two possible responses according to the position of

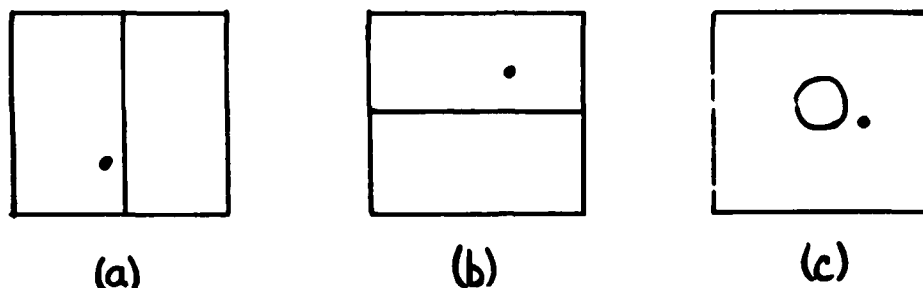


Figure 8 Information Processing Task with Switching

the dot relative to the threshold displayed: left or right (a), up or down (b), inside or outside (c). Responses are registered by depressing one of two mechanical, horizontally-arranged, buttons. Dot position is determined independently of threshold type, and the relative mixture of threshold types is controlled by the experimenter. For this task, a processing time model is desired.

#### Switching Model

Because of the physical response mechanism arrangement relative to the threshold types, the subject will presumably have to re-orient his association of dot positions to response buttons each time the threshold type changes. If one considers that a comparison test for each threshold type is accomplished using a procedure, the task in Figure 8 is one that will in general require switching among procedures. Thus the model suggested earlier can be tested on this task. In particular, if the characteristics of each individual threshold procedure are assumed fixed, then the general switching model of eq.(29) specializes to

$$T_p = p_h \cdot \mu_h + \alpha_h \cdot H(p_h) + p_v \cdot \mu_v + \alpha_v \cdot H(p_v) + p_c \cdot \mu_c + \alpha_c \cdot H(p_c) \quad (34)$$

where  $\mu$ ,  $p$ , and  $\alpha$  have been used as a shorthand for  $\bar{t}_{p1}$ ,  $p(i)$ , and  $\alpha_i$ , respectively, and the subscripts v, h, and c designate vertical, horizontal, and circular thresholds, respectively.

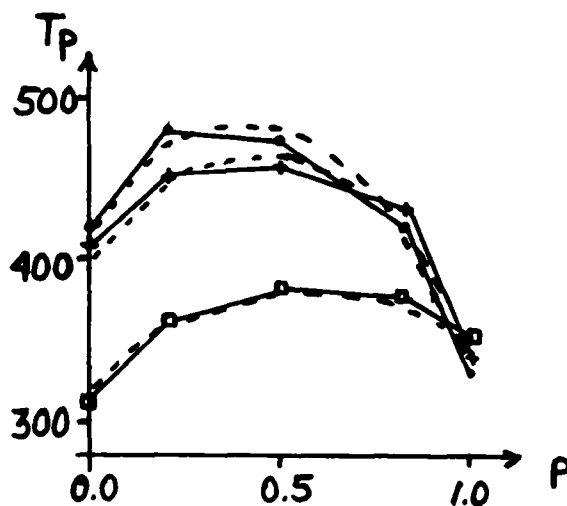
### 3.2.2 Test of Model

#### Setup and General Procedure

The task shown in Figure 8 was implemented similarly to the single procedure task discussed earlier. A 20 unit ( $\approx 6$  inches) square border was continuously displayed on a CRT, with the coordinate (0,0) at the center of the display. Dot position (horizontal and vertical) was generated on each trial using values drawn from a  $N(0,2)$  distribution. Circle radius was 2.5 units. An experimental run consisted of a pre-announced number of trials (usually 100 or 200) for which a particular set of  $p_v$ ,  $p_h$ ,  $p_c$  values was selected to establish the experimental condition. Subjects were not informed as to the threshold mixture except to state which threshold types would be included in the mixture. In other words, the subject was informed that the run would be "all verticals", "horizontals and circles", "a 3-way mixture", etc. On each trial the threshold and dot would appear simultaneously, and the subject was instructed to respond as quickly as possible, but with certainty. An 800 ms second blanking interval was used between trials. The dot display was actually a 0.1 unit diameter circle, and positions were adjusted, if necessary, so that the dot and threshold never intersected.

#### Experimental Tests and Results

While the primary experimental goal in the present case was to investigate the validity of the model in eq.(34), results from several preliminary experimental sessions also lend support to the switching model. In these sessions, subjects were systematically tested at conditions of 2-way threshold mixtures, one session each for the three possible binary combinations. Experimental runs of 200 trials were used. The results for one subject are shown in Figure 9, where average processing time  $T_p$  is plotted versus the parameter  $p$ , which designates the mixture. Conditions are labeled in the figure according to the types of thresholds in the binary mixture. The interpretation of  $p$  for that condition is also given. For example, "hv" designates the binary



Cond	Symb	p
hv	□	p(h)
hc	+	p(h)
vc	•	p(v)

Subject: RF

Figure 9 Binary Switching Results

combination of horizontals and verticals, and  $p$  in this case denotes the fraction of horizontals, i.e.  $p = p(h)$ . Five mixtures for each binary combination were tested. It is evident from the figure that an overhead in processing was required for switching from one threshold type to another. Table 4 contains parameter estimates for models of each

Table 4 Parameter Estimates for Binary Switching Data

Cond	$\mu_h$	$\mu_v$	$\mu_c$	$\hat{\alpha}_h + \hat{\alpha}_v$	$\hat{\alpha}_h + \hat{\alpha}_c$	$\hat{\alpha}_v + \hat{\alpha}_c$	95% Conf Interval
hv	342	310	-	48	-	-	$\pm 10$
hc	339	-	397	-	81	-	$\pm 13$
vc	-	325	411	-	-	103	$\pm 13.5$

threshold combination.

The identified model for each condition has been superimposed on the observed data in Figure 9. Note that because of the symmetry of  $M(\cdot)$  it is not possible to uniquely estimate  $\alpha$  values for two-way mixture data. Therefore, only a sum has been estimated. A 95% confidence interval for this sum is also given in Table 4. The results in Figure 9 and Table 4 indicate that the task is one that requires measurable switching time, and

that the switching model proposed provides a reasonable characterization of the observed data. Similar results were obtained from other subjects.

A test of the general 3-way switching model was conducted as follows. Figure 10 show the simplex of possible 3-way threshold mixtures. For a

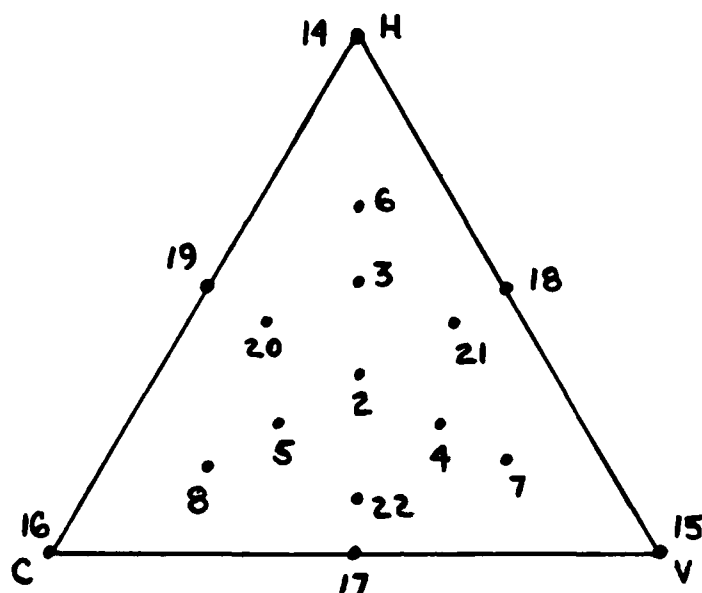


Figure 10 Simplex of 3-Way Mixtures

given point in the simplex, the corresponding  $p_H$ ,  $p_V$ , and  $p_C$  values are obtained by measuring the normalized distance of the point to each side. For example, point 3 corresponds to  $p_H$ ,  $p_V$ , and  $p_C$  values of 0.5, 0.25, and 0.25, respectively. In a single session, subjects were given a set of mixtures chosen from various points in this simplex. The results from some conditions in the set were used to estimate model parameters, and the identified model was then used to predict the results for the remaining conditions in the set. In particular, for the results to be shown later, the conditions for estimating model parameters were selected as

$$\{14-16, 17-19, 6-8\} \quad (35)$$

and conditions for model test were selected from among elements of the set

$$\{2, 5, 20-22\} \quad (36)$$



Conditions 14-16 correspond to exclusive use of one threshold type; conditions 17-19 are binary mixtures where each threshold type used is equally likely; and conditions 6-8 are 3-way mixtures where one type occurs on 2/3 of the trials and the other two types are equally likely to occur on the rest of the trials.

The conditions selected for model parameter estimation were chosen to be deliberately distinct in the simplex from those used for model testing. This was done in order to also test the ability of the model to predict, to some extent, behavior outside the region from which it was constructed. Thus the test conditions tend toward the "middle" of the simplex, while the calibration conditions are positioned near the "edges."

The experimental procedure for two subjects was to conduct 2 runs of 100 trials at each condition, and to use the second run in data calculations. The subjects were aware that mixtures were occurring in identical pairs of 100 trials each, but they were not told that only the second run would be of primary interest. Presumably, the first run would allow the subject to make any adjustments to the mixture characteristics, and the second run would represent a consistent sequence of responses at that particular mixture. A third subject (RF) was tested using experimental runs of length 100 for conditions 14-16 and 17-19, but runs of length 200 for all other conditions. For all conditions involving RF, however, there was a preliminary sequence of 100 so that adjustment to mixture characteristics could be made.

Subjects were not aware which conditions were being used for model calibration and which for test. Furthermore, the calibration and test conditions were interspersed with each other. Subjects generally performed the task at a consistent level of accuracy; error rates were typically between 1 and 3 percent.

Estimates for model parameters  $\mu$  were obtained directly from observed data; that is, the mean value of processing time observed was taken as the

value of  $\mu$ . Estimates for the parameters  $\alpha$  were obtained using a least squares method. The resulting models are tabulated by subject and session in Table 5, along with simultaneous 95% confidence intervals on the

Table 5 Model Parameter Estimates for 3-Way Switching

Session	Subject	$\hat{\mu}_h$	$\hat{\mu}_v$	$\hat{\mu}_c$	$\hat{\alpha}_h$	$\hat{\alpha}_v$	$\hat{\alpha}_c$	95% Conf for $\alpha$
1	BM	289	246	340	3.9	32.3	52	$\pm 18$
2	BM	273	257	320	16.4	29.8	19.6	$\pm 18$
3	PP	309	294	370	29.4	24.5	46.9	$\pm 15$
4	PP	284	272	350	3.4	23.1	40.7	$\pm 19$
5	RF	310	310	357	37.8	14.3	29.3	$\pm 7$

estimates of  $\alpha$  values.

Using the identified models for each respective session predicted values of processing time were obtained for other conditions, along with 95% confidence intervals. These predictions are shown in Table 6 together with the actual processing times observed for the respective conditions.

### Discussion

There are several points of interest with respect to the results displayed in Tables 5 and 6. First, considerable variation from session to session is evident, both across and within subjects. Observed  $\mu$  values do tend to be consistent across subjects, however. Their ordering is the same, and for two subjects  $\hat{\mu}_h$  is only slightly more than  $\hat{\mu}_v$ . Processing time for circular threshold trials is much higher by comparison. The estimated values of  $\alpha$  do not exhibit any particular pattern, although there does appear to be some tendency for  $\hat{\alpha}_h$  to be the smallest of the three. The confidence intervals for  $\alpha$  values indicate that the overhead for switching is significantly greater than zero.

There was observed to be a significant difference in subject alertness from session to session. In Sessions 1 and 4, the subjects were

Table 6 Switching Model Predictions and Test Results

Sess	Subj	Cond	T <sub>p</sub>	$\hat{T}_p$	95%	Sess	Subj	Cond	T <sub>p</sub>	$\hat{T}_p$	95%
1	BM	20	382	378	±13	3	PP	20	424	423	± 11
		21	330	339	"			21	377	388	"
		22	355	378	"			22	406	415	"
		2	370	370	±12			2	450	415	± 10
2	BM	21	315	328	±16	4		20	373	368	± 8
		22	353	344	"			21	352	334	"
		2	330	342	±13			22	372	374	"
		3	357	337	"			2	388	362	± 9
						5	RF	5	399	405	±10
								2	390	399	"

more alert by comparison to Sessions 2 and 3, respectively. However, though it is interesting to compare results across sessions, the test of the switching model was made with single session data. For this purpose, it has been assumed that subjects were at a consistent attention level for the entire session (which lasted less than 2 hours).

In examining the predicted vs. observed processing time values of Table 7, there is substantial agreement between model and data. More than 75% of the tests have  $T_p$  within the 95% confidence bounds; in many cases, values of  $T_p$  are much closer to  $\hat{T}_p$ . There does not seem to be any condition where a systematic violation occurs.

As mentioned earlier, the test conditions were chosen deliberately to be somewhat different or removed in the simplex from the model calibration conditions. Given this underlying strategy, along with the relatively few data points taken at each condition, the agreement evident in Table 7 is encouraging. While issues pertaining to variability across subjects and sessions can legitimately be raised, the conclusion from this set of

experiments is that switching among procedures clearly takes a significant amount of additional time and that the switching model of eq.(29) is a promising descriptive approach to accounting for this effect in a processing time model.

#### IV. SUMMARY

This paper has suggested an approach to describing human processing time as a function of task variables. An essential feature of the approach is to view humans not as limited capacity channels, but rather as limited capacity systems. This has led to the generalization of the classical information theoretic processing time model. In particular, a correspondence was made between systems of variables and mental processing procedures. Based on this association, it was suggested that a model for processing time could be developed by effectively projecting processing time onto a basis of variables that were postulated as representative of intermediate processing steps. To facilitate the argument, single variable systems were cited as an important special case. Specifically, there is a direct correspondence between error-free channels and single variable systems; thus, the model developed using the generalized information theoretic framework is identical with that developed using the classical limited capacity channel approach.

In addition to single procedure models, the approach suggested in this paper has been extended to include situations where multiple procedures are used in an alternating fashion. A specific model form for the additional processing time required for switching among procedures has emerged from information theoretic considerations. Like the basic procedure model form, switching overhead is determined from experimental observation.

To test the modeling approach, two sets of experiments have been devised and executed. In the first, the task situation was such that it would presumably be accomplished using a single procedure. A model was

developed in terms of task variables using the approach discussed in this paper and parameters were estimated using experimentally observed results. The identified model was then used to predict results for other experimental conditions, with mixed success. On the whole, however, there was in the observed results evidence that supports the procedure model, which in turn lends credibility to the procedure-based modeling approach.

In the second set of experiments, a task that focused on procedure switching was considered. As with the first set of experiments, a model was developed and parameters were estimated from experimental results. The identified model was then used to predict results at other experimental conditions. There was observed to be substantial agreement between predicted and observed processing time, which offers considerable support for the specific model and modeling approach as a description of processing time required to switch among procedures.

Although the results of the experimental tests indicate that the procedure-based modeling approach has promise, the conclusion is by no means final. Additional testing and development is necessary. In particular, the general situation where procedures and switching characteristics are allowed to vary simultaneously should be examined as a logical next step. In addition, it may also be worthwhile to consider technical aspects of the definition of a procedure's internal variables in order to improve the descriptive power of the approach. Both of these directions for additional investigation serve to underscore the intent of this paper, which is to describe a particular modeling approach in its early stages of development and to offer some evidence in support of its major features. A good deal of further development is necessary in order to establish the approach on a firm basis.

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